

BIPOLAR INTERVAL VALUED INTUITIONISTIC FUZZY NECESSITY OPERATOR

M.Suganya^[1], A.Manonmani^[2]

^[1]Research Scholar, Department of Mathematics, LRG Government Arts college for Women,
Tirupur.

^[2]Assistant Professor, Department of Mathematics, LRG Government Arts college for Women,
Tirupur.

Abstract

In this paper we have introduced the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified its property.

Keyword

Bipolar Interval Valued Intuitionistic Fuzzy Topological Space, Bipolar Interval Valued Intuitionistic Fuzzy Set.

1. Introduction:

Lee introduced the concept of Bipolar fuzzy set. In Bipolar Intuitionistic Fuzzy Topology the membership and non-membership degree of the fuzzy set lies in the range $[0,1]$ and $[-1,0]$ [21]. In this paper we have introduced the Bipolar Interval Valued Intuitionistic Fuzzy necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset of a Bipolar Interval Valued Intuitionistic Fuzzy Topological space and verified that the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy Subset itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topological space.

2. Definition:

Let X be a non-empty set, and let A be a Bipolar interval valued intuitionistic fuzzy set on a Bipolar Interval Valued Intuitionistic Topological Space $BIVIFTS(X)$, then the necessity operator on A is defined as

$$i. \quad [A] = \left\{ \left\langle x, \left[\begin{array}{cc} \mu^P_{AL}(x), \mu^P_{AU}(x) \\ \mu^N_{AL}(x), \mu^N_{AU}(x) \end{array} \right], \left[\begin{array}{cc} 1 - \mu^P_{AL}(x), 1 - \mu^P_{AU}(x) \\ 1 + \mu^N_{AL}(x), 1 + \mu^N_{AU}(x) \end{array} \right] \right\rangle \mid x \in X \right\}$$

2.1. Theorem:

Let $(X, \mathcal{I})$ be a Bipolar Interval Valued Intuitionistic Fuzzy Topological Space (BIVIFTS). Based on the necessity operator on a Bipolar Interval Valued Intuitionistic Fuzzy set A on X , we can also construct several BIVIFTSs on X as

$$\mathcal{I}_N = \{ [A] \mid A \in \mathcal{I} \}$$

i.e., the necessity operator defined in the above definition itself forms a topology.

Proof:

In order to prove the topology we have to prove the following

Let S be a set and $\mathcal{I}$ be a family of bipolar interval valued intuitionistic fuzzy subset of S . The family is called a Bipolar Interval Valued Intuitionistic Fuzzy Topology (BIVIFT) on S if $\mathcal{I}$ satisfies the following axioms

- i. $0_s, 1_s \in \mathcal{I}$
- ii. If $\{A_i; i \in I\} \subseteq \mathcal{I}$, then $\bigcap_{i=1}^{\infty} A_i \in \mathcal{I}$
- iii. If $A_1, A_2, A_3, \dots, A_n \in \mathcal{I}$, then $\bigcap_{i=1}^n A_i \in \mathcal{I}$

Let $A_1, A_2, \dots, A_i$ be Bipolar interval valued intuitionistic fuzzy subsets on a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X).

To prove necessity operator is a Bipolar Interval Valued Intuitionistic Topological Space BIVIFTS(X)

- i. obviously $0_s, 1_s \in \mathcal{I}_N$
- ii.

$$A \sqcap B = \left\langle \left[x, \mathcal{I}_{(A \sqcap B)L}^p(x), \mathcal{I}_{(A \sqcap B)U}^p(x) \right], \left[\mathcal{I}_{(A \sqcap B)L}^N(x), \mathcal{I}_{(A \sqcap B)U}^N(x) \right] \right\rangle \mid x \in X$$

$$\left\langle \left[\mathcal{I}_{(A \sqcap B)L}^p(x), \mathcal{I}_{(A \sqcap B)U}^p(x) \right], \left[\mathcal{I}_{(A \sqcap B)L}^N(x), \mathcal{I}_{(A \sqcap B)U}^N(x) \right] \right\rangle$$

where

$$\mathcal{I}_{(A \sqcap B)L}^p(x) = \min \{ \mathcal{I}_A^p(x), \mathcal{I}_B^p(x) \}$$

$$\begin{aligned} \mathbb{I}_{(A \sqcup B)U}^P(x) &= \max \left\{ \mathbb{I}_{AU}^P(x), \mathbb{I}_{BU}^P(x) \right\} \\ \mathbb{I}_{(A \sqcup B)L}^N(x) &= \max \left\{ \mathbb{I}_{AL}^N(x), \mathbb{I}_{BL}^N(x) \right\} \\ \mathbb{I}_{(A \sqcup B)U}^N(x) &= \min \left\{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{BU}^N(x) \right\} \\ \mathbb{I}_{(A \sqcup B)L}^P(x) &= \min \left\{ \mathbb{I}_{AL}^P(x), \mathbb{I}_{BL}^P(x) \right\} \\ \mathbb{I}_{(A \sqcup B)U}^P(x) &= \max \left\{ \mathbb{I}_{AU}^P(x), \mathbb{I}_{BU}^P(x) \right\} \\ \mathbb{I}_{(A \sqcup B)L}^N(x) &= \max \left\{ \mathbb{I}_{AL}^N(x), \mathbb{I}_{BL}^N(x) \right\} \\ \mathbb{I}_{(A \sqcup B)U}^N(x) &= \min \left\{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{BU}^N(x) \right\} \end{aligned}$$

$$\mathbb{I}_{A_1 \sqcup A_2}^P(x) = \max \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\}, \mathbb{I}_{A_1 \sqcup A_2}^N(x) = \min \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\}$$

where

$$\begin{aligned} \mathbb{I}_{(A_1 \sqcup A_2)L}^P(x) &= \min \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\} \\ \mathbb{I}_{(A_1 \sqcup A_2)U}^P(x) &= \max \left\{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x) \right\} \\ \mathbb{I}_{(A_1 \sqcup A_2)L}^N(x) &= \max \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\} \\ \mathbb{I}_{(A_1 \sqcup A_2)U}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \\ \mathbb{I}_{(A_1 \sqcup A_2)L}^P(x) &= \min \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\} \\ \mathbb{I}_{(A_1 \sqcup A_2)U}^P(x) &= \max \left\{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x) \right\} \\ \mathbb{I}_{(A_1 \sqcup A_2)L}^N(x) &= \max \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\} \\ \mathbb{I}_{(A_1 \sqcup A_2)U}^N(x) &= \min \left\{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \right\} \end{aligned}$$

then

$$\begin{aligned} \mathbb{I}_{(A_1 \sqcup A_2)L}^P(x) &= \min \left\{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \right\} \\ \mathbb{I}_{(A_1 \sqcup A_2)U}^P(x) &= \max \left\{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x) \right\} \\ \mathbb{I}_{(A_1 \sqcup A_2)L}^N(x) &= \max \left\{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \right\} \end{aligned}$$

$$\begin{aligned}
 \mathbb{I}_{([A_1] \sqcup [A_2])U}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2])L}^P(x) &= \min \{ 1 \sqcup \mathbb{I}_{A_1L}^P(x), 1 \sqcup \mathbb{I}_{A_2L}^P(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2])U}^P(x) &= \max \{ 1 \sqcup \mathbb{I}_{A_1U}^P(x), 1 \sqcup \mathbb{I}_{A_2U}^P(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2])L}^N(x) &= \max \{ 1 \sqcup \mathbb{I}_{A_1L}^N(x), 1 \sqcup \mathbb{I}_{A_2L}^N(x) \} \\
 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2])U}^N(x) &= \min \{ 1 \sqcup \mathbb{I}_{A_1U}^N(x), 1 \sqcup \mathbb{I}_{A_2U}^N(x) \} \\
 [A_1] \sqcup [A_2] &= \left\langle \begin{aligned} & \left[\mathbb{I}_{([A_1] \sqcup [A_2])L}^N(x), \mathbb{I}_{([A_1] \sqcup [A_2])U}^N(x) \right], \\ & \left[1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2])L}^P(x), 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2])U}^P(x) \right], \\ & \left[1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2])L}^N(x), 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2])U}^N(x) \right] \end{aligned} \right\rangle | x \in X \\
 [A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i] &= \left\langle \begin{aligned} & \left[\mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])L}^N(x), \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}^N(x) \right], \\ & \left[1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])L}^P(x), 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}^P(x) \right], \\ & \left[1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])L}^N(x), 1 \sqcup \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}^N(x) \right] \end{aligned} \right\rangle | x \in X
 \end{aligned}$$

where

$$\begin{aligned}
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])L}^P(x) &= \min \{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x), \dots, \mathbb{I}_{A_iL}^P(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}^P(x) &= \max \{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x), \dots, \mathbb{I}_{A_iU}^P(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])L}^N(x) &= \max \{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x), \dots, \mathbb{I}_{A_iL}^N(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x), \dots, \mathbb{I}_{A_iU}^N(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])L}^P(x) &= \min \{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x), \dots, \mathbb{I}_{A_iL}^P(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}^P(x) &= \max \{ \mathbb{I}_{A_1U}^P(x), \mathbb{I}_{A_2U}^P(x), \dots, \mathbb{I}_{A_iU}^P(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])L}^N(x) &= \max \{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x), \dots, \mathbb{I}_{A_iL}^N(x) \} \\
 \mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x), \dots, \mathbb{I}_{A_iU}^N(x) \}
 \end{aligned}$$

then

$$\mathbb{I}_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])L}^P(x) = \min \{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x), \dots, \mathbb{I}_{A_iL}^P(x) \}$$

Hence the arbitrary union of Bipolar Interval Valued Intuitionistic Necessity
ors is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

$$A \sqcap B = \left\langle \left[x, \vartheta_{(A \sqcap B)L}^p(x), \vartheta_{(A \sqcap B)U}^p(x) \right], \left[\vartheta_{(A \sqcap B)L}^N(x), \vartheta_{(A \sqcap B)U}^N(x) \right], \left[\vartheta_{(A \sqcap B)L}^p(x), \vartheta_{(A \sqcap B)U}^p(x) \right], \left[\vartheta_{(A \sqcap B)L}^N(x), \vartheta_{(A \sqcap B)U}^N(x) \right] \right\rangle \mid x \sqsubseteq X$$
$$\begin{aligned} \mathbb{Q}^P_{(A \sqcup B)L}(x) &= \max \left\{ \mathbb{Q}^P_{AL}(x), \mathbb{Q}^P_{BL}(x) \right\} \\ \mathbb{Q}^P_{(A \sqcup B)U}(x) &= \min \left\{ \mathbb{Q}^P_{AU}(x), \mathbb{Q}^P_{BU}(x) \right\} \\ \mathbb{Q}^N_{(A \sqcup B)L}(x) &= \min \left\{ \mathbb{Q}^N_{AL}(x), \mathbb{Q}^N_{BL}(x) \right\} \\ \mathbb{Q}^N_{(A \sqcup B)U}(x) &= \max \left\{ \mathbb{Q}^N_{AU}(x), \mathbb{Q}^N_{BU}(x) \right\} \\ \mathbb{Q}^P_{(A \sqcup B)L}(x) &= \max \left\{ \mathbb{Q}^P_{-}(x), \mathbb{Q}^P_{-}(x) \right\} \\ \mathbb{Q}^P_{(A \sqcup B)U}(x) &= \min \left\{ \mathbb{Q}^P_{-}(x), \mathbb{Q}^P_{-}(x) \right\} \\ \mathbb{Q}^N_{(A \sqcup B)L}(x) &= \min \left\{ \mathbb{Q}^N_{-}(x), \mathbb{Q}^N_{-}(x) \right\} \end{aligned}$$

$$\mathbb{I}_{(A \sqcup B)U}^N(x) = \max \{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{BU}^N(x) \}$$

then

$$(\mathbb{I}_1^A \sqcap \mathbb{I}_2^A) = \left\langle \begin{array}{l} \left[\mathbb{I}_{A_1 \sqcup A_2}^N(x), \mathbb{I}_{A_1 \sqcup A_2}^N(x) \right], \\ \left[\mathbb{I}_{A_1 \sqcup A_2}^N(x), \mathbb{I}_{A_1 \sqcup A_2}^N(x) \right], \\ \left[\mathbb{I}_{A_1 \sqcup A_2}^P(x), \mathbb{I}_{A_1 \sqcup A_2}^P(x) \right], \\ \left[\mathbb{I}_{A_1 \sqcup A_2}^N(x), \mathbb{I}_{A_1 \sqcup A_2}^N(x) \right] \end{array} \right\rangle | x \in X$$

where

$$\begin{aligned} \mathbb{I}_{A_1 \sqcup A_2}^P(x) &= \max \{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \max \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^P(x) &= \max \{ \mathbb{I}_{A_1L}^P(x), \mathbb{I}_{A_2L}^P(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \{ \mathbb{I}_{A_1L}^N(x), \mathbb{I}_{A_2L}^N(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \max \{ \mathbb{I}_{A_1U}^N(x), \mathbb{I}_{A_2U}^N(x) \} \end{aligned}$$

then

$$\begin{aligned} \mathbb{I}_{A_1 \sqcup A_2}^P(x) &= \max \{ \mathbb{I}_{AL}^P(x), \mathbb{I}_{AL}^P(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{AU}^N(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \{ \mathbb{I}_{AL}^N(x), \mathbb{I}_{AL}^N(x) \} \\ \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \max \{ \mathbb{I}_{AU}^N(x), \mathbb{I}_{AU}^N(x) \} \\ 1 \sqcap \mathbb{I}_{A_1 \sqcup A_2}^P(x) &= \max \{ 1 \sqcap \mathbb{I}_{AL}^P(x), 1 \sqcap \mathbb{I}_{AL}^P(x) \} \\ 1 \sqcap \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \{ 1 \sqcap \mathbb{I}_{AU}^N(x), 1 \sqcap \mathbb{I}_{AU}^N(x) \} \\ 1 \sqcap \mathbb{I}_{A_1 \sqcup A_2}^N(x) &= \min \{ 1 \sqcap \mathbb{I}_{AL}^N(x), 1 \sqcap \mathbb{I}_{AL}^N(x) \} \end{aligned}$$

$$1 \square \square^N_{([]_{A_1 \square []_{A_2}})_U} (x) = \max \{ 1 \square \square^N_{A_1 U} (x), 1 \square \square^N_{A_2 U} (x) \}$$

$$\square [] A_1 \square [] A_2 = \left(\begin{array}{l} x, \square^P_{([]_{A_1 \square []_{A_2}})_L} (x), \square^P_{([]_{A_1 \square []_{A_2}})_U} (x),, \\ \square^N_{([]_{A_1 \square []_{A_2}})_L} (x), \square^N_{([]_{A_1 \square []_{A_2}})_U} (x),, \\ 1 \square \square^P_{([]_{A_1 \square []_{A_2}})_L} (x), 1 \square \square^P_{([]_{A_1 \square []_{A_2}})_U} (x),, \\ 1 \square \square^N_{([]_{A_1 \square []_{A_2}})_L} (x), 1 \square \square^N_{([]_{A_1 \square []_{A_2}})_U} (x), \end{array} \right) | x \square X \square^N$$

$$\square [] A_1 \square [] A_2 \square \dots [] A_i = \left(\begin{array}{l} x, \square^P_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_L} (x), \square^P_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_U} (x), \\ \square^N_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_L} (x), \square^N_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_U} (x), \\ \square^P_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_L} (x), \square^P_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_U} (x), \\ \square^N_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_L} (x), \square^N_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_U} (x), \end{array} \right) | x \square X \square^N$$

where

$$\square^P_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_L} (x) = \max \{ \square^P_{[]_{A_1 L}} (x), \square^P_{[]_{A_2 L}}, \dots, \square^P_{[]_{A_i L}} (x) \}$$

$$\square^P_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_U} (x) = \min \{ \square^P_{[]_{A_1 U}} (x), \square^P_{[]_{A_2 U}}, \dots, \square^P_{[]_{A_i U}} (x) \}$$

$$\square^N_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_L} (x) = \min \{ \square^N_{[]_{A_1 L}} (x), \square^N_{[]_{A_2 L}}, \dots, \square^N_{[]_{A_i L}} (x) \}$$

$$\square^N_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_U} (x) = \max \{ \square^N_{[]_{A_1 U}} (x), \square^N_{[]_{A_2 U}}, \dots, \square^N_{[]_{A_i U}} (x) \}$$

$$\square^P_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_L} (x) = \max \{ \square^P_{[]_{A_1 L}} (x), \square^P_{[]_{A_2 L}}, \dots, \square^P_{[]_{A_i L}} (x) \}$$

$$\square^P_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_U} (x) = \min \{ \square^P_{[]_{A_1 U}} (x), \square^P_{[]_{A_2 U}}, \dots, \square^P_{[]_{A_i U}} (x) \}$$

$$\square^N_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_L} (x) = \min \{ \square^N_{[]_{A_1 L}} (x), \square^N_{[]_{A_2 L}}, \dots, \square^N_{[]_{A_i L}} (x) \}$$

$$\square^N_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_U} (x) = \max \{ \square^N_{[]_{A_1 U}} (x), \square^N_{[]_{A_2 U}}, \dots, \square^N_{[]_{A_i U}} (x) \}$$

then

$$\square^P_{([]_{A_1 \square []_{A_2} \square \dots []_{A_i}})_L} (x) = \max \{ \square^P_{A_1 L} (x), \square^P_{A_2 L}, \dots, \square^P_{A_i L} (x) \}$$

$$\begin{aligned}
 \mathbb{I}^P_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])U}(x) &= \min \left\{ \mathbb{I}^P_{A_1U}(x), \mathbb{I}^P_{A_2U}(x), \dots, \mathbb{I}^P_{A_iU}(x) \right\} \\
 \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x) &= \min \left\{ \mathbb{I}^N_{A_1L}(x), \mathbb{I}^N_{A_2L}(x), \dots, \mathbb{I}^N_{A_iL}(x) \right\} \\
 \mathbb{I}^N_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}(x) &= \max \left\{ \mathbb{I}^N_{A_1U}(x), \mathbb{I}^N_{A_2U}(x), \dots, \mathbb{I}^N_{A_iU}(x) \right\} \\
 1 \sqcap \mathbb{I}^P_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x) &= \max \left\{ 1 \sqcap \mathbb{I}^P_{A_1L}(x), 1 \sqcap \mathbb{I}^P_{A_2L}(x), \dots, 1 \sqcap \mathbb{I}^P_{A_iL}(x) \right\} \\
 1 \sqcap \mathbb{I}^P_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}(x) &= \min \left\{ 1 \sqcap \mathbb{I}^P_{A_1U}(x), 1 \sqcap \mathbb{I}^P_{A_2U}(x), \dots, 1 \sqcap \mathbb{I}^P_{A_iU}(x) \right\} \\
 1 \sqcap \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x) &= \min \left\{ 1 \sqcap \mathbb{I}^N_{A_1L}(x), 1 \sqcap \mathbb{I}^N_{A_2L}(x), \dots, 1 \sqcap \mathbb{I}^N_{A_iL}(x) \right\} \\
 1 \sqcap \mathbb{I}^N_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}(x) &= \max \left\{ 1 \sqcap \mathbb{I}^N_{A_1U}(x), 1 \sqcap \mathbb{I}^N_{A_2U}(x), \dots, 1 \sqcap \mathbb{I}^N_{A_iU}(x) \right\}
 \end{aligned}$$

$$\left[\bigcap_{i=1}^n A_i \right] = \left\langle \begin{array}{l} x, \mathbb{I}^P_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x), \mathbb{I}^P_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}(x), \\ \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x), \mathbb{I}^N_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}(x), \\ 1 \sqcap \mathbb{I}^P_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x), 1 \sqcap \mathbb{I}^P_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}(x), \\ 1 \sqcap \mathbb{I}^N_{([A_1] \sqcap [A_2] \sqcap \dots \sqcap [A_i])L}(x), 1 \sqcap \mathbb{I}^N_{([A_1] \sqcup [A_2] \sqcup \dots \sqcup [A_i])U}(x) \end{array} \right\rangle | x \in X$$

Hence the finite intersection of Bipolar Interval Valued Intuitionistic Necessity Operators is in Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

Hence the Bipolar Interval Valued Intuitionistic Necessity Operators itself forms a Bipolar Interval Valued Intuitionistic Fuzzy Topology τ .

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